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- USSR -

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FOREWORD

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PROPAGATION OF ELASTIC-PLASTIC WAVES DURING COMPOUND LOADING

USSR

[Following is a translation of an article by N. Cristescu in Prikladnaya matematika mekhanika (Applied Mathematics and Mechanics), Vol XXIII, No. 6, 1959, pages 1124-1128.]

In Reference (1) Kh. A. Rakhmatulin considered some problems on the propagation of elastic-plastic waves under conditions of compound loading. He examined the case when the wave of compound loading is one producing pronounced rupture, propagating with a velocity below that of ordinary elastic-plastic waves (Reimann waves). Thus, Rakhmatulin assumes that in the case of compound loading during impact, a group of ordinary plastic waves is at first propagated in a plastic body, followed by one causing pronounced rupture, the wave of compound loading.

The present work examines the same problem on the basis of equations established by Rakhmatulin, and at the same time considers other possible cases of propagation which can exist for certain materials. For example, it is shown that for such materials compound dynamic loading is generally transmitted in a plastic body only by compound waves. These waves propagate in the body faster than ordinary plastic waves (2, 3). The investigation was more qualitative, since the theory of small elastic-plastic deformations, employed in the present work, has not yet been checked experimentally or properly adapted for dynamic problems.

Let us examine the problem posed by Rakhmatulin on the compressing-shearing collision of two free bands. The material of the bands is considered to be elastic-plastic and to satisfy the equations in the theory of small elastic-plastic deformations. Performing certain physically valid simplifications, Rakhmatulin reduces these equations to the form of the equations (1.5) shown in Reference (1).

$$\frac{2}{3} X_x - \frac{1}{3} Y_y = \frac{2}{3} \frac{1}{e_i} \left(\frac{2}{3} e_{xx} - \frac{1}{3} e_{zz}\right)$$

$$-\frac{1}{3} (X_x + Y_y) = \frac{2}{3} \frac{1}{e_i} \left(\frac{2}{3} e_{zz} - \frac{1}{3} e_{xx}\right)$$

$$\frac{1}{3}(X_x + Y_y) = k\,(e_{xx} + e_{zz})$$

$$e_i = \frac{2}{3}\sqrt{\left(\frac{u}{x}\right)^2 + \left(\frac{u}{x} - e_{zz}\right)^2 + e_{zz}^2 + \frac{3}{2}\left(\frac{v}{x}\right)^2} \qquad (1)$$

The intensity of stress $_i$ is a function of the intensity of deformation

$$_i = \; _i\,(e_i) \qquad (2)$$

To equations (1) and (2) we must add the equations of motion, which can be written in the form

$$\frac{^2u}{t^2} = \frac{X_x}{x}, \qquad \frac{^2v}{t^2} = \frac{Y_x}{x} \qquad (3)$$

since it is assumed that the motion components u and v depend only on one space coordinate x and on the time t:

$$u = u\,(x, t), \qquad v = v\,(x, t) \qquad (4)$$

In all the above formulas, the components are "average" components and were so denoted by Rakhmatulin by the index ^{o}. For simplicity, this index is omitted.

In the future, for easier calculation we shall assume that the material is incompressible.

$$e_{xx} + e_{zz} = 0 \qquad (5)$$

Applying the condition of incompressibility (5), we reduce system (1) to the form

$$2X_x - Y_y = 2\,\frac{_i}{e_i}\,e_{xx}, \qquad X_x + Y_y = 2\,\frac{_i}{e_i}\,e_{xx}$$
$$e_i = \frac{1}{3}\sqrt{4\left(\frac{u}{x}\right)^2 + \left(\frac{v}{x}\right)^2} \qquad (6)$$

where $e_{xx} = u/ x$ and $e_{xy} = v/ x$ is the component of deformation.

The stress component can be expressed as a function of deformation components in the following manner:

$$X_x = \frac{4}{3}\frac{_i}{e_i}\frac{u}{x}, \qquad Y_y = \frac{2}{3}\frac{_i}{e_i}\frac{u}{x}, \qquad Y_x = \frac{1}{3}\frac{_i}{e_i}\frac{v}{x} \qquad (7)$$

We shall assume that the function contained in (2) increases monotonically and has a concave curve tending toward the axis O_{ei} (in particular, it can be an exponential function, for example).

$$\frac{{}^2u}{t^2} = L\frac{{}^2u}{x^2} \neq 4N\frac{{}^2v}{x^2}, \qquad \frac{{}^2v}{t^2} = M\frac{{}^2v}{x^2} \neq N\frac{{}^2u}{x^2} \tag{8}$$

where

$$L = \frac{4}{3}\frac{i}{e_i} \neq \frac{4}{3e_i^3}\left(\frac{u}{x}\right)^2(e_i \quad i' - i) \qquad N = \frac{4}{9e_i^3}\frac{u}{x}\frac{v}{x}(e_i \quad i' - i)$$

$$M = \frac{1}{3}\frac{i}{e_i} \neq \frac{1}{3e_i^3}\left(\frac{v}{x}\right)^2(e_i \quad i' - i) \tag{9}$$

Consequently, using expressions (2) and (7), the functions L, M, and N can be expressed simply by $u/\ x$ and $v/\ x$.

Equations (8) are equations of motion, corresponding to equations (1.6) in Reference (1). From (8) it follows that for the problem under study, in the general case two types of waves occur. We shall make the usual assumption that, in passing through a given front, the increment waves,

$$du_x = \frac{u_x}{x}dx \neq \frac{u_x}{t}dt, \qquad du_t = \frac{u_t}{x}dx \neq \frac{u_t}{t}dt$$

$$dv_x = \frac{v_x}{x}dx \neq \frac{v_x}{t}dt, \qquad dv_t = \frac{v_t}{x}dx \neq \frac{v_t}{t}\ t$$

taken along this front, remain continuous; to equations (8) we shall add

$$\frac{{}^2u}{x^2}dx^2 - \frac{{}^2u}{t^2}dt^2 = du_xdx - du_tdt, \quad \frac{{}^2v}{x^2}dx^2 - \frac{d^2v}{dt^2}dt^2 = dv_xdx - dv_tdt \tag{10}$$

Having solved the system consisting of equations (8) and (10), with respect to the highest-order derivatives, it is easy to obtain the characteristic equations of the system

$$p^2\left(\frac{dx}{dt}\right)^4 - p\,(L + M)\left(\frac{dx}{dt}\right)^2 + LM - 4N^2 = 0 \tag{11}$$

and the differential expressions which are satisfied in the characteristics. There are four of these expressions, but, taking into account (11), only one of them is independent; for example, the expression

$$(c\,du_x - du_t)N + (pc^2 - {}'L)\,(c\,dv_x - dv_t) = 0 \tag{12}$$

Here, C is the velocity of propagation of the plastic wave front, in which the expression (12) is satisfied. As follows from (11), the velocity can have two values:

$$\left.\begin{array}{l} c_I^2 = c_I^2\left(\dfrac{\partial u}{\partial x}, \dfrac{\partial v}{\partial x}\right) \\ c_{II}^2 = c_{II}^2\left(\dfrac{\partial u}{\partial x}, \dfrac{\partial v}{\partial x}\right) \end{array}\right\} = \left(\frac{dx}{dt}\right)^2 = \frac{L + M \pm \sqrt{(L - M)^2 + 16N^2}}{2p} \tag{13}$$

Since, obviously, $L > 0$ and $M > 0$, it therefore follows

$$c_I > c_{II}$$

The velocities c_I and c_{II}, determined from the relationship (13), are velocities of the propagation of two types of waves of complex loading. We shall designate these types of waves (I) and (II), respectively. In particular, in regions where it is possible to consider $v = 0$ the waves will propagate with the velocity

$$c_{I1}^2 = \frac{4}{3}\,\frac{\sigma_{i1}'}{p} \tag{15}$$

of a Riemann wave (I1); here (σ'_{i1}) is calculated from the expression formally resembling (2)

$$\sigma_{i1} = \sigma_{i1}\,(e_{i1})$$

however, in reality those expressions are dissimilar.

Finally, in regions where it is possible to consider $u = 0$, there will propagate only Riemann waves (II 2) with a velocity

$$c_{II2}^2 = \frac{1}{3} \frac{\sigma_{i2}'}{p} \tag{16}$$

Here σ_{i2}' is calculated from another expression of the $\sigma_{i2} = \sigma_{i2}(e_{i2})$ type.

We shall show that under some conditions, i.e., with certain mechanical properties of the plastic material,

$$c_I^2 > \frac{L}{P} > c_{I_1}^2 \tag{17}$$

The first part of the inequality (17) follows directly from (13); for proving the second part it is necessary to compare the expression L from the relationship (9) with $c_{I_1}^2$ from (15), i.e., we must have

$$\frac{1}{3}\left[4\frac{\sigma_i}{e_i} + \frac{16}{3e_i^3}\left(\frac{\partial u}{\partial x}\right)^2 (e_i\sigma_i' - \sigma_i)\right] > \frac{4}{3}\sigma_{i1}'$$

or (18)

$$\frac{1}{3e_i^2}\left(\frac{\partial v}{\partial x}\right)^2 \left(\frac{\sigma_i}{e_i} - \sigma_i'\right) + \sigma_i' > \sigma_{i1}'$$

The inequality (18) is satisfied for very many types of materials, for which ordinarily $\sigma_i/e_i \gg \sigma_i'$, and σ_i' does not differ too greatly from $_{i1}'$.

From (17) we can make the important conclusion that, in materials satisfying the foregoing conditions, waves of compound loading (I) propagate faster than ordinary Riemann plastic waves (I1). Indeed, if we compare the velocity c_I with the velocity $c_{I1elast}$ of elastic waves, we will obtain $c_I < c_{I1elast}$, although in some cases these two velocities are almost equal. Consequently, for instantaneous compound loading at the end of a plastic body it must be considered that wave (I) is that causing pronounced rupture.

This reasoning is not valid for the velocity c_{II}, because from (13) it easily follows that $c_{II}2 < M/p$, and comparing M/p with $c_{II_2}{}^2$, we obtain $c_{II_2}{}^2 < M/p$ and, consequently, in this way c_{II} and c_{II_2} cannot be compared. Directly comparing c_{II} and c_{II_2}, and making the same assumptions as above, it can be shown that

$$c_{II_2}{}^2 < c_{II}{}^2 \qquad (19)$$

and hence, waves (II 2) propagate slower than waves (II).

We shall now show that waves (I) and (II) are actually waves of compound loading, and not ordinary plastic waves. We shall denote by α and β the discontinuities in the derivatives $\partial^2 u/\partial x^2$ and $\partial^2 v/\partial x^2$ during passage of the front of the waves

$$\alpha = [\frac{\partial^2 u}{\partial x^2}] = \frac{\partial^2 u}{\partial x^2}\Big|_{+} - \frac{\partial^2 u}{\partial x^2}\Big|_{-}, \quad \beta = [\frac{\partial^2 v}{\partial x^2}] = \frac{\partial^2 v}{\partial x^2}\Big|_{+} - \frac{\partial^2 v}{\partial x^2}\Big|_{-}$$

If this front is that of wave (I), we shall denote these by α_I and β_I, and if of the type (II) wave, by α_{II} and β_{II}. From (10) it follows that, between discontinuities of second-order derivatives, there exist the relationships

$$[\frac{\partial^2 u}{\partial x^2}]\, c^2 - [\frac{\partial^2 u}{\partial t^2}] = 0, \quad [\frac{\partial^2 v}{\partial x^2}]\, c^2 - [\frac{\partial^2 v}{\partial t^2}] = 0 \qquad (20)$$

while from (8), if we also bear in mind (20), we obtain the expression

$$(pc^2 - L)\alpha - 4N\beta = 0, \quad N\alpha - (pc^2 - M)\beta = 0 \qquad (21)$$

The equations (21) are independent, since from (11) it follows

$$\frac{pc^2 - L}{N} = \frac{4N}{pc^2 - M} \qquad (22)$$

Equations (21) and (12) also are independent, if we keep in mind the relationship (10).

Consequently, in wave front (I) we have

$$(\rho c_I^2 - L)\alpha_I - 4N\beta_I = 0 \tag{23}$$

and in wave front (II) we have

$$(\rho c_{II}^2 - L)\alpha_{II} - 4N\beta_{II} = 0 \tag{24}$$

From (23) and (24) it follows that, in both wave front I and wave front I [sic, means II,-Trand.] all second-order derivatives of u and v are discontinuous. Consequently, both waves are waves of compound loading. These waves of compound loading degenerate into ordinary plastic waves only when $N = 0$. This can occur in two cases.

In the first case, one of the deformation components is equal to zero. Consequently, $\partial u/\partial x = 0$ or $\partial v/\partial x = 0$. In this case, system (8) is reduced to a single equation: to the second or first equation of (8), respectively (where $N = 0$). The differential relationship (12), satisfied in the characteristics, is reduced to one of the known relationships:

$$dv_t = c_{II2} dv_x \qquad \text{or} \qquad du_t = c_{I1}\, du_x$$

Consequently, in this case, in a plastic body only one type of ordinary wave is propagated; the velocity of propagation is determined, respectively, by relationship (16) or (15).

The second case, when $N = 0$, is the case of an elastic body ($\sigma_i/e_i = \sigma_i'$). In this case, system (8) is reduced to two ordinary equations for the propagation of two types of elastic waves, and the velocities of propagation reduce to known constant velocities. The discontinuities in these two ordinary wave fronts are independent in the sense that they are not linked by any relationship [for instance, the type of relationship (12)] and propagate at different velocities.

From the above it follows that compound loading propagates in a body by two groups of ordinary waves, if the body remains elastic, i.e., during impact the elastic limit is not reached. When the elastic limit is exceeded, plastic deformations propagate in the body by two types of waves of compound loading, whose velocity of propagation is determined by the relationship (13). Preceding these waves, elastic waves can, generally speaking, also be propagated. In any case, during instantaneous compound loading ordinary plastic waves do not arise in the body, since waves of compound loading, arising simultaneously at the end of the band, propagate faster.

If impact loading is not compound loading in the sense that different deformation components do not develop simultaneously but arise gradually, and if loading is not instantaneous, then two types of ordinary plastic waves will be propagated in a plastic body. Depending on marginal conditions, the possibility exists for the simultaneous propagation of two types of ordinary plastic waves in certain regions of the plastic body. Nevertheless, in this case plastic waves of compound loading do not appear, because discontinuities in the derivative $\partial^2 u / \partial x^2$ are distributed at a velocity different from that of the discontinuity in the derivative $\partial^2 v / \partial x^2$, and no relationship exists between these discontinuities. Thus, these waves propagate independently, although the wave which propagates faster makes the body inhomogeneous, and in this sense influences the succeeding wave.

We shall now calculate the discontinuities, assuming that in the plastic body both wave I and wave II are being propagated. Let us assume that the two fronts of waves I and II simultaneously pass a certain cross section x_0 at a moment of time t_0. We shall note that the discontinuities in the two wave fronts are not independent, since if we bear in mind the relationship $p(c_I^2 + c_{II}^2) = L + M$, then equations (23) and (24) can be written in the form

$$\frac{2N}{pc_I^2 - L} = \frac{\alpha_I}{2\beta_I} = -\frac{2\beta_{II}}{\alpha_{II}} \qquad (25)$$

where N, L and c_I are calculated for $x = x_0$ and $t = t_0$. The total discontinuity α^* in the derivative $\partial^2 u // \partial x^2$ during passage of the two wave fronts is the sum of the two discontinuities $\alpha^* = \alpha_I + \alpha_{II}$. Similarly, for the total discontinuity in the derivative $\partial^2 v / \partial x^2$ we obtain

$$\beta^* = \beta_I + \beta_{II}$$

The coefficients of the discontinuities always satisfy the relationship

$$\alpha_I \alpha_{II} - 4\beta_I \beta_{II} = 0 \qquad (26)$$

If we know the total discontinuities α^* and β^* at a certain cross section x_0 (in particular, $x_0 = 0$) and moment of time t_0, and also the stressed state, then from (25) we obtain the discontinuities in each wave individually:

$$\alpha_{I} = \frac{\alpha^*}{2} + \beta^*\omega, \quad \beta_{I} = \tfrac{1}{2}\left(\frac{\alpha^*}{2} + \beta^*\right), \quad \alpha_{II} = \frac{\alpha^*}{2} - \beta^* \tag{27}$$

$$\beta_{II} = \tfrac{1}{2}\left(\beta^* - \frac{\alpha^*}{2\omega}\right) \quad \omega = \frac{2N}{pc_I^2 - L}$$

It should be noted that not all discontinuities have the same sign.

For approximate solution of the problem many methods can be proposed.

If loading is instantaneous, we shall then assume that waves I and II are waves causing pronounced rupture, propagating at constant velocities (13). Then, in the plane x0t (see figure) we have four regions: region 1 is not deformed, region 2 is elastically deformed, and regions 3 and 4 are plastically deformed after passage of the waves of compound loading I and II, respectively.

In region 2 the solution is known from solution of the elastic problem. Consequently, in this region the values are known for the deformation and the velocities u_{x2}, u_{t2}, v_{x2}, v_{t2}. In region 3 they are determined by the expressions

$$\begin{aligned} v_{t3} + c_1 v_{x3} &= v_{t2} + c_1 v_{x2}, \quad c_I p(v_{t3} - v_{t2}) = - X_{y3} + X_{y2} \\ u_{t3} + c_1 u_{x3} &= u_{t2} + c_1 u_{x2}, \quad c_I p(u_{t3} - u_{t2}) = - X_{x3} + X_{x2} \end{aligned} \tag{28}$$

to which we shall add the first expression in (13) for determining c_I, as well as expression (7).

The transition from region 3 to region 4 is accomplished similarly.

If the loading at $x = 0$ can be considered a succession of instantaneous loadings, then it is possible to proceed exactly in the same manner, only making the plane xt include a larger number of regions.

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